Shortest Path Planning Under Turn-Radius Constraints

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Introduction

Optimal path planning with turn constraints is computationally difficult. Existing methods have limited practical utility on graphs with non-uniform edges because they do not account for edge length. This allows planners to admit sequences of edges that can satisfy local angular checks but form kinematically infeasible maneuvers. In high resolution environments the likelihood of such maneuvers increases as edges shrink in length.

Geometric Solutions

Enforcing physical maneuverability requires incorporating metric length into the path validation process. A vehicle's turn radius $R_{\min}$ is directly related to its maximum allowable curvature by the relationship $\kappa_{max} = \frac{1}{R_{min}}$. We decompose this continuous constraint into two discrete parameters: a maximum turn angle $\theta_{\max}$, and a minimum length scale $L_{\min}$. By requiring every turn to span at least distance $L_{\min}$ with a heading change $\leq \theta_{\max}$, we discretely approximate bounded-curvature motion.

In practice, we can enforce this globally by ensuring individual edges satisfy a first-order threshold of $L_{\min}/2$. Because every turn involves at least two edges (approach and departure) this "half-length" requirement mathematically guarantees that the full maneuver distance $L_{\min}$ is met. This simple adjustment transforms a complex pairwise check into an efficient, local constraint on individual edges.

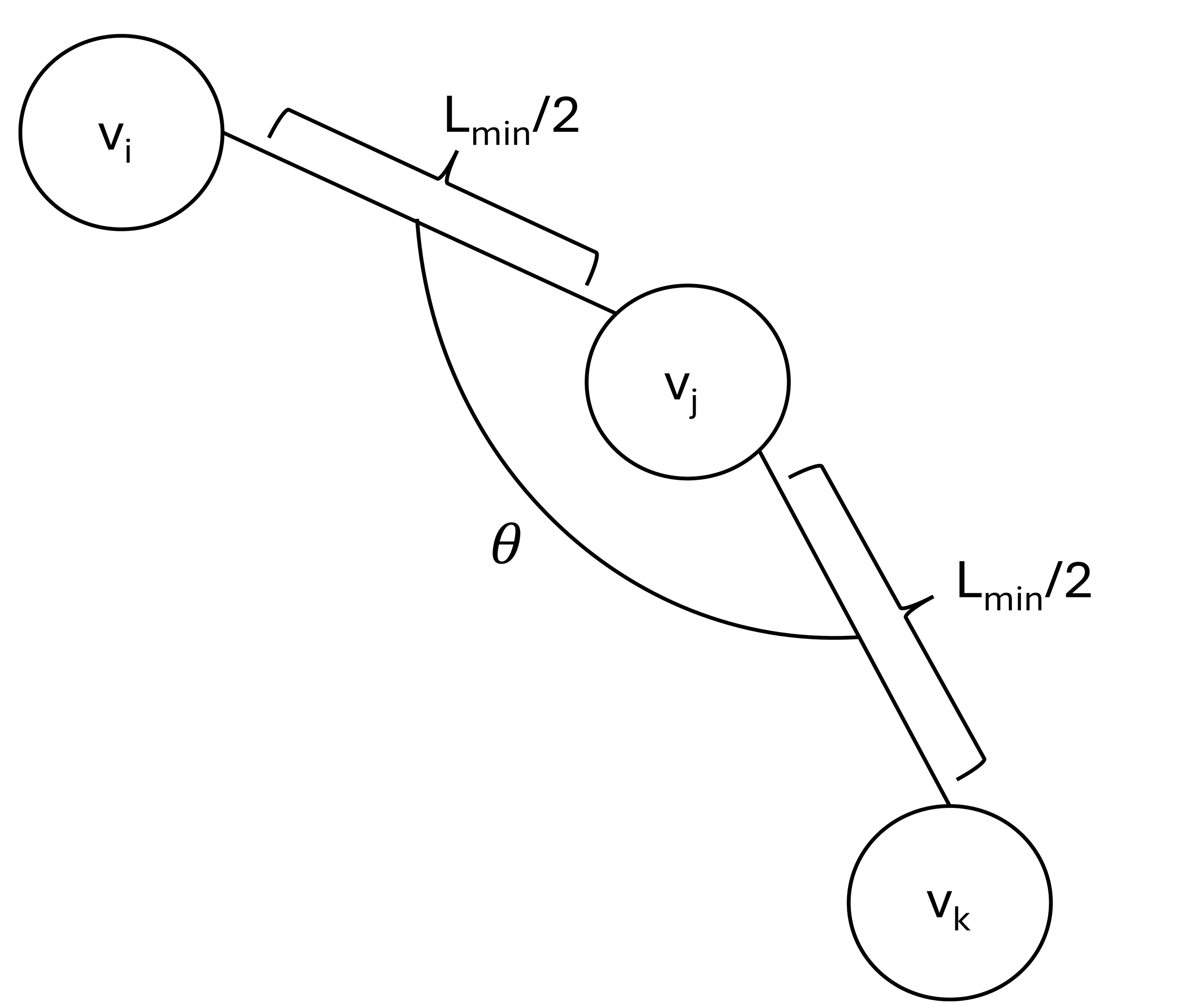

Figure 1: Length-scale Enforcement Diagram

Enforcement

Enforcing both angle and length constraints transforms turn validation into a 2D query over edge angle and length. We introduce two equivalent enforcement strategies:

Static Filtering
Edges shorter than $L_{\min}/2$ are removed during preprocessing. This guarantees all remaining edge sequences satisfy the length requirement, allowing local angle checks to ensure global feasibility at runtime.

Dynamic Runtime Filtering
Constraints are enforced during search using a Priority Search Tree (PST), which directly returns only edges satisfying both angle and length bounds, enabling flexible use across heterogeneous platforms.

Both paradigms enforce the same underlying constraints and therefore produce identical optimal paths. The distinction lies in when enforcement occurs: preprocessing for maximum runtime efficiency for one platform versus query-time filtering for heterogeneous fleets.

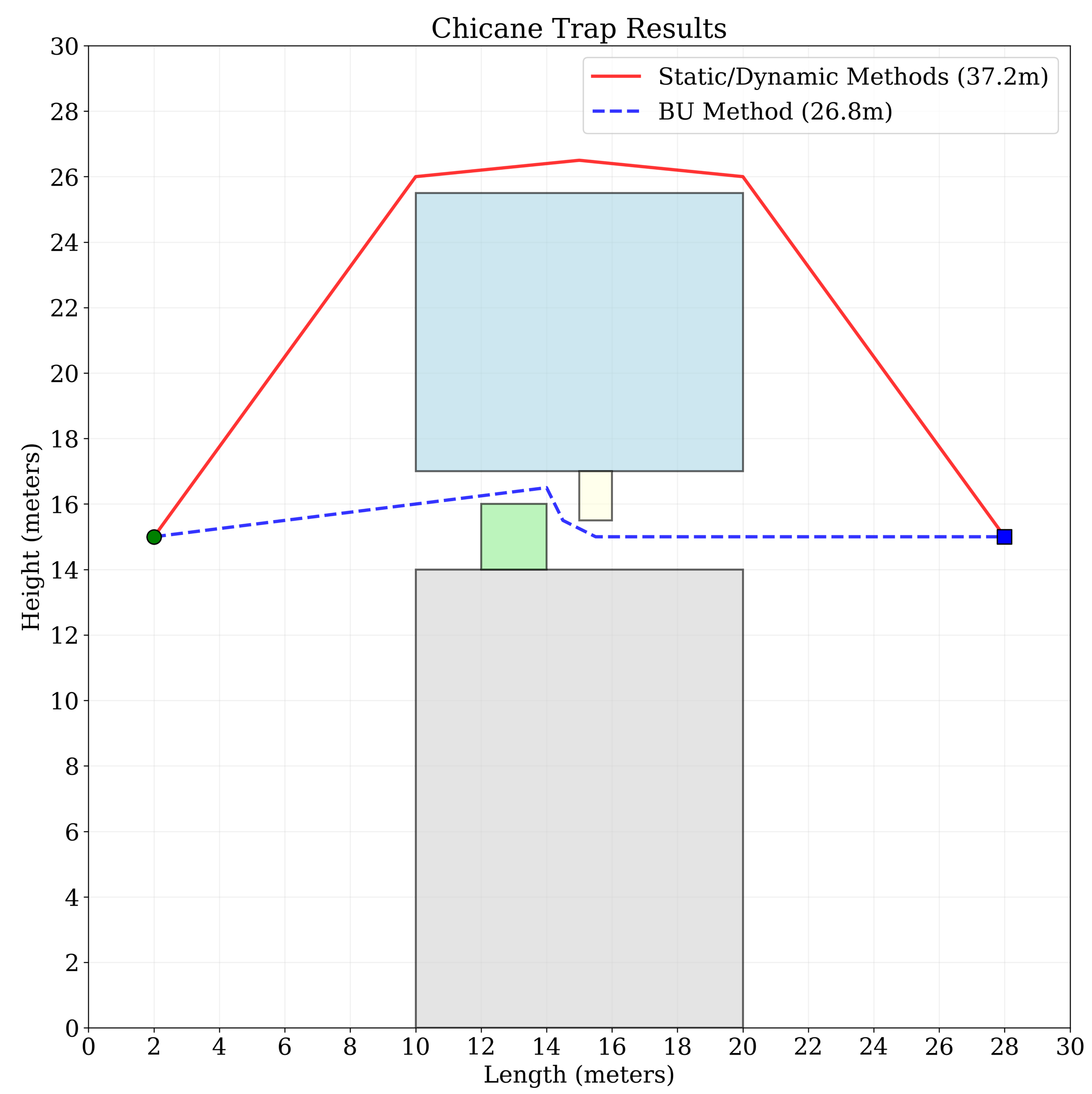

Figure 2: “Chicane Trap” Results

The static and dynamic methods (red) produce identical paths, demonstrating their equivalence under joint angle and length constraints. By incorporating length awareness, the method avoids infeasible micro-maneuvers and instead selects a longer but physically realizable trajectory. In contrast, the baseline angle-only method (blue) pursues the absolute shortest path, admitting an impossible maneuver due to its lack of metric (length) information.

Figure 3: Benchmarks of proposed methods

Search performance was evaluated across varying grid resolutions (0.5 to 4.0 cells/m) to compare enforcement paradigms against the Boroujerdi & Uhlmann (BU) baseline. The Static Graph approach demonstrates near-identical runtime to the unconstrained baseline, confirming that embedding the $L_{\min}/2$ requirement into the graph topology during preprocessing effectively eliminates runtime overhead.

In the online paradigms, the Dynamic Linear enforcement exhibits a significant performance penalty as resolution increases due to “candidate explosion”. The search must individually evaluate numerous angularly valid edges that fail the length constraint. The Dynamic PST method avoids this performance degradation due to its true orthogonal range queries. This structure enables the retrieval of all valid edges in $O(\log n + k_{\text{valid}})$ time, maintaining performance that closely tracks both the static and BU baseline.

Conclusion

We presented a framework for optimal shortest path planning under turn-radius constraints. By enforcing a minimum length scale $L_{\min}$ and a maximum turn angle ($\theta_{\max}$), our method guarantees valid maneuvers and eliminates discretization-dependent false positives.

This approach bridges the gap between discrete graph search and continuous kinodynamic planning while maintaining the $O(|E|\log|V|)$ complexity of previous vertex-only methods. Ultimately, these paradigms provide a scalable, resolution-independent solution for reliable autonomous navigation on arbitrary geometric graphs.